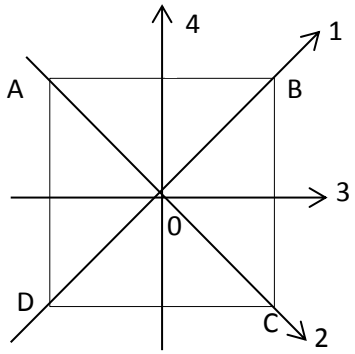
Assignments on Rotational Dynamics

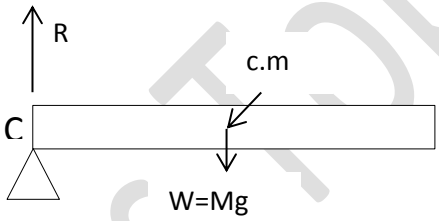
1. A flywheel has a moment of inertia 0.05 kg m^2. What constant unbalanced torque is required to increase its speed from 60 rpm to 300 rpm in 8s? [0.157 n-m]
2. Go-round, 20m in diameter. Find his angular momentum if the merry-go-round is making 6 rpm. [2.512 X 10^3 kg m^2/s]
3. (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with frequency of 40 rpm. How much is the frequency of revolution of the child if he folds his hands back and thereby reduce his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.
   (b) Prove that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy? [100 rpm]
4. A stone of mass m is tied to the end of a string is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then the tension in the string is given by $T=Ar^n$ where A is constant, r is the instantaneous radius of the circle and n is a number. Find n. [n=-3]
5. Select the correct alternative: A mass m is moving with a constant velocity along a line parallel to the x-axis from the origin. Its angular momentum with respect to the origin (a) is zero, (b) remains constant, (c) goes on increasing, (d) goes on decreasing. [(b)]
6. a disc has moment of inertia 0.04 kg m^3. Find the work required to increase its speed from 60 rpm to 150 rpm. [4.15 J]
7. Three point masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of length a. calculate the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1. $[\frac{a^2}{4}(m_2+m_3)]$
8. a rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 0.4 m. what is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping. [25 rad s^{-2},10 m s^{-2}]

9. Choose the correct alternative(s): the moment of inertia of a thin square plate ABCD (see fig.) of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is (A) $I_1+ I_2$, (B) $I_3+ I_4$, (C) $I_1+ I_3$, (D) $I_1+ I_2+ I_3 +I_4$ where I_1, I_2, I_3 and I_4 are respectively the moments of inertia about axes 1, 2, 3 and 4 which are in the plane of the plate.



[(A),(B) and (C)]

10. A uniform rod of weight W is supported horizontally by two vertical supports at its ends. At a certain instant one of the supports is suddenly removed. What is the reaction at the other support at that instant? [W/4]

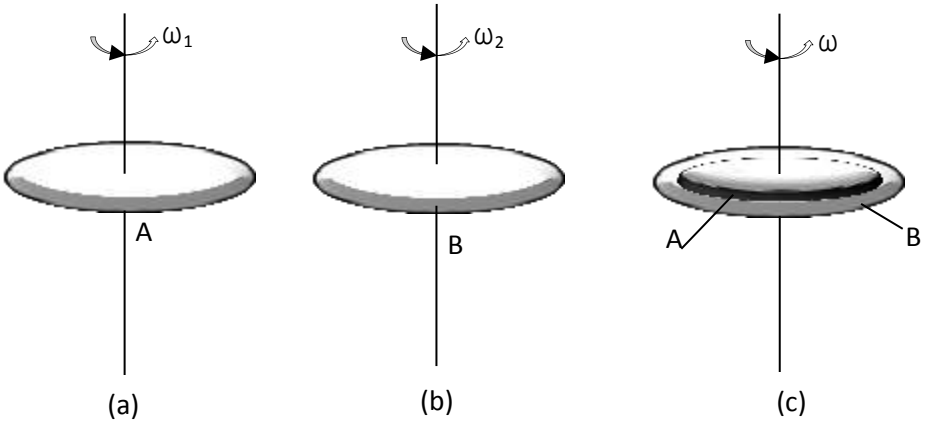


11. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre) and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(a) What is the angular velocity of the two-disc system?

(b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss energy? Given $\omega_1 \neq \omega_2$.

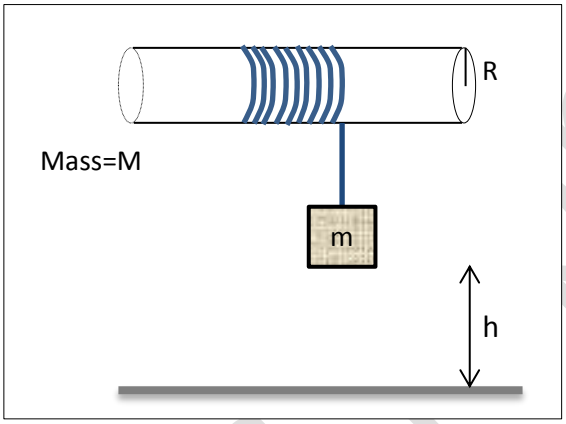
$$\left[\frac{I_1\omega_1+ I_2\omega_2}{I_1+ I_2}, \frac{1}{2}\frac{I_1 I_2}{I_1+ I_2}(\omega_1 - \omega_2)^2\right]$$



(a) (b) (c)

12. A cylinder rolls down an inclined plane of inclination θ. Show that the cylinder will slip if μ, the coefficient of friction is less than $\frac{1}{3}\tan\theta$.

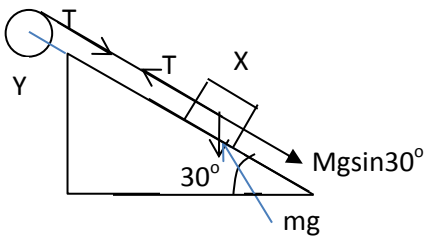
13. A light flexible rope is wound round a solid cylinder of mass M and radius R. The rope carries at its free end a mass m which is at rest at a height h above the floor. The mass m is now released. When it hits the floor, what is the angular velocity of the cylinder? Assume friction to be absent and the cylinder to rotate about its own axis. Also find out the tension in the rope and the accelaration of mass m.



$\left[\omega=\frac{1}{R}\sqrt{\frac{2gh}{1+M/2m}}, T=\frac{mMg}{2m+M}\right]$

14. A carpet of mass M of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligible small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2. $[v=\sqrt{\frac{14}{3}gr}]$

15. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30^o to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in the figure. The drum is given an initial angular velocity such that the block X starts moving up the plane.



(a) Find the tension in the string during the motion.

(b) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest. [1.634 N, 1.224m]

16. A solid sphere of mass 1 kg and radius 0.10 m is rotating uniformly about a diameter with angular velocity π radians/second. Using standard formula calculate its kinetic energy. [19.74 X 10^{-3} J]

17. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by distance d and are rotating about centre of mass, which is stationary. Find the ratio of the angular momentum of star B about the centre of mass. [Ans. 6]

18. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of this lamina about axes passing through O and P is I_0 and I_P, respectively. Both these axes are perpendicular to the plane of lamina. Find the ratio $\frac{I_P}{I_O}$ to the nearest integer.

[Ans. 3]