Projectile Motion

1. The trajectory of a projectile in a vertical plane is $y= ax-bx^2$, where a, b are constants and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. Evaluate the maximum height attained and the angle of projection from the horizontal.
2. A ball is dropped from a tower. 2 seconds later another ball is dropped. Taking g = $9.8m/s^2$ find the relative velocity of the first ball with respect to the second.
3. A ball is thrown from the ground level to strike a vertical wall at a horizontal distance x away at a height y above the ground when travelling horizontally, show that the velocity of projection is $[g(2y+\frac{x^2}{2y})]^{1/2}$.
4. A particle is thrown from a tower of height H with an initial velocity u, if θ_{max} is the angle for which we have a maximum range R_{max}, then show that $\theta_{max} = Cot^{-1}[R_{max}\ g / u^2\ ]$ with, $R_{max}= \frac{u}{g}\sqrt{2gH+u^2}$.
5. A ball is thrown vertically upward with an initial velocity u, it crosses a height h in time t_1 and t_2, while going up and coming down, show that $t_1 + t_2 = \frac{2u}{g}$, which is independent of h. Hence show that, time taken to reach the maximum height is u/g.
6. A particle is projected at an angle α to the horizon, so as just to clear to equal walls of equal height 'a', at a distance '2a' from each other. Show that the horizontal range for the projectile is $2a \cot\frac{\alpha}{2}$.
7. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls at the other end of the base. If α and β be the base angles and θ the angle of projection, show that $\tan\theta=\tan\alpha +\tan\beta$.
8. At what angle should a body be projected with a velocity $24ms^{-1}$ just to pass over the obstacle 16 m high at a horizontal distance of 32 m? consider g= $10\ ms^{-2}$.
9. A fighter plane flying horizontally at an altitude of 1.5 km with a speed 720 kmh^{-1} passes directly overhead an aircraft gun. At what angle from the vertical should the gun be fired for the shell nuzzles speed 600 ms^{-1} to hit the plane? At what maximum altitude should the pilot fly the plane to avoid being hit? consider g= $10\ ms^{-2}$.
10. A bomber, flying upwards at an angle of 53^o with the vertical and releases a bomb at an altitude of 800 m. the bomb strikes the ground 20 s after its release. Find
    (i) The velocity of the bomber at the time of release of the bomb,
    (ii) The maximum height attained by the bomb,
    (iii) The horizontal distance travelled by the bomb before it strikes the ground.
    (iv) The velocity (magnitude and direction) of the bomb just when it strikes the ground.
    Take $\sin 53^o =0.8$, $\cos 53^o= 0.6$, g= $10\ ms^{-2}$

11. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30^{o}$ and the angle of the barrel to the horizontal $\beta = 60^{o}$. The initial velocity u of the shell is 21 ms^{-1}. Evaluate the distance from the gun to the point at which the shell falls.
12. A ball rolls off the top of a staircase with a constant horizontal velocity u. If the steps are h metre high and w metre wide, show that the ball will just hit the edge of nth step if $n=\frac{2hu^2}{gw^2}$.