Assignment :: Vector

1. Find component of vector A + B along (i) x-axis, (ii) C.
   Given $A = \hat{\imath} - 2\hat{\jmath}$, $B = 2\hat{\imath} + 3\hat{k}$ and $C = \hat{\imath} + \hat{\jmath}$.
2. If $A = 2\hat{\imath} - 3\hat{\jmath} + 7\hat{k}$, $B = \hat{\imath} + 2\hat{\jmath}$ and $C = \hat{\jmath} - \hat{k}$. Find A . (B x C).
3. In a triangle ABC, M is the mid-point of the side AB. The line segments CM and MB represent the vectors P and Q respectively. Express each of the following directed line segments in terms of vectors P and Q (i) CB, (ii) AM, (iii) MA, (iv) AB, (v) CA.
4. Two forces of 6×10^{-4} N and 8×10^{-4} N acting at an angle of 60° with each other, pull on an object. What single pull would replace the given forces?
5. The resultant of two vectors 3P and 2P is R. If the first vector is doubled, the resultant vector also becomes double. Find the angle between the vectors.
6. What is the resultant of a displacement 4 km N and one 10 km E?
7. What is the angle between two equal forces so that their resultant is one-third of one of the forces?
8. At take off, a helicopter rises 15 meter while moving north 8 meter and west 6 meter. How far is it from its starting position?
9. Prove vectorially that the line joining the middle points of two adjacent sides of a triangle is parallel to and half of the third side.
10. The maximum and minimum values of the resultant of two forces acting at a point are 15 N and 7 N respectively. If the magnitude of both the forces are increased by 1 N and if these two new forces act at an angle of 90°, find the magnitude and direction of the resultant.
11. A boat travels 8 km/hr in still water. If the velocity of the water current is 4 km/hr, at what angle with the shore must the boat be steered to reach a point directly on the opposite bank?
12. The larger one of two vectors is double the smaller. Prove that the angle which their resultant makes with the larger one can't be greater than 30°.
13. The maximum value of the magnitude of the resultant of two vectors P and Q (P > Q) is a times its least value. If the angle between the two vectors be θ and the magnitude of the resultant be half the sum of the magnitudes of the two vectors, then show that $\cos\theta = \frac{a^2+2}{2(1-a^2)}$.

14. Three forces can be represented by the three medians of a triangle. Prove that these three forces remain in equilibrium (i.e., their resultant is zero).
15. The resultant of two forces P and Q inclined at a definite angle is R and the angle between R and P is θ. Prove that if the forces (P + R) and Q be inclined at the same definite angle, then their resultant makes an angle θ/ 2 with (P + R).
16. A boy pulls a rope attached to a box with a force of 3×10^{-4} N. The rope makes an angle of 60^o with the ground. Compute the effective value of the pull tending to move the box along the ground and the effective value tending to lift the box vertically.
17. A force of 3×10^{-4} N is inclined at an angle of 60^o with the y-axis. Determine the components of the force along x- and y-axes.
18. A man can reach just the opposite point of the bank by swimming in time t_1 and can swim the same distance down the current in time t_2. If the speed of the man in still water be u and that of current be v, then find the ratio of t_1 and t_2.
19. Compute the resultant of the following system of coplanar forces the angles being given with respect to the x-axis ; 3×10^{-4} N at 0^o ; 4×10^{-4} N at 30^o ; 4×10^{-4} N at 150^o.
20. Find the magnitude of the vector $r_1 = 5i - 6j + 8k$. Find also the magnitude and direction of the resultant of this vector and the vector $r_2 = 20i + 6j – 8k$.
21. If the co-ordinates of the terminus of a vector be (2,-4, -5), then represent the vector and find its magnitude.
22. Prove that the following three vectors $\overrightarrow{AB}$ = 3 i - 4 j - 3 k, $\overrightarrow{BC}$ = - i - 2 j - 3 k and

    $\overrightarrow{AC}$ = 2 i – 6 j - 6 k form a triangle.
23. Find the magnitude of the vector $3\hat{\imath} + 4\hat{\jmath} + 12\hat{k}$ and the angles it makes with the X, Y and Z axes.
24. Vectors A and B are represented as follows :

$$A = 10i - 12j + 5k$$

$$B = 7i + 8j - 12k$$

i, j and k are unit vectors along x, y and z axes respectively. Express the resultant vector and calculate its magnitude.

25. The co-ordinates of A and B are respectively (-1, 5, 7) and (3, 2, –5). Represent the vector $\overrightarrow{AB}$ by these co-ordinates.
26. Prove that A . (A x B) = 0
27. Prove that $|A \times B|^2 = |A|^2 |B|^2 - (A . B)^2$.
28. If |A + B| = |A – B| show that A is perpendicular to B.
29. Show that the magnitude of the vector product of two vectors gives numerically the area of the parallelogram formed with those two vectors as adjacent sides.
30. If a = i + 2j + 4k and b = 2i + j + 8k, find a . b, a x b and b x a.
31. If A = 3i + 3j - 3k and B = 2i + j + 3k, find the angle between A and B.
32. If $A = A_x i + A_y j$ and $B = B_x i + B_y j$, determine the condition that (i) they are perpendicular to each other, (ii) they are parallel to each other.
33. Calculate the area of a parallelogram whose two adjacent sides are given by $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$. Also find the unit vector along the normal to the plane of $\vec{A}$ and $\vec{B}$.
34. Prove that for a triangle

$$\frac{sinA}{a} = \frac{sin\,B}{b} = \frac{sinC}{c}$$

where a, b, c are three sides and A, B, C are the three opposite angles.

35. Can two forces of magnitude 6 kg and 10 kg respectively have a resultant of 12 kg? What are the greatest and least values that the magnitude of their resultant can have? Explain.
36. A vector r is the resultant of two vectors a and b which make angles of 30^o and 60^o respectively with it on opposite sides. How large are the latter vectors?
37. Add the following forces by the component method; 3×10^{-4} N at 30^o, 1.5×10^{-4} N at 120^o and 1.0×10^{-4} N at 315^o to x-axis.
38. Two vectors 2P and P are inclined to each other at certain angle. If the first vector is doubled, then the resultant vector is increased three times. Calculate the angle between the two vectors.
39. Two vectors A and B of equal magnitudes are at right angle to each other. Prove that the vectors (A + B) and (A - B) have equal magnitudes and are normal to each other.

40. The resultant of two vectors $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ is $\vec{P}$. If the direction of $\overrightarrow{F_2}$ is reversed, the resultant is $\vec{Q}$. Show that $(P^2 + Q^2) = 2(F_1^2 + F_2^2)$.

41. The angle between two vectors $\vec{P}$ and $\vec{Q}$ is θ. Show by resolving these vectors into rectangular components that the magnitude of their resultant is $(P^2 + Q^2 + 2PQ\cos\theta)^{1/2}$.

42. If A = -4i + 2j + 4k and B = 2i + 4j + 4k, find the angle between A and B.

43. If A = 4i + 5j + 6k and B = 6i + 5j + 4k, find A . B, A x B and B x A.

44. $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x- and y-axes respectively. What is the magnitude and direction of the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$? What are the components of a vector $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ along the direction $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$?

45. If the resultant of two forces is equal in magnitude to one of them and perpendicular to it in direction, find the other.

46. The resultant of two forces P and Q acting at a point is equal to $\sqrt{3}$Q and makes an angle 30^o with the direction of P. Show that either P = Q or P = 2Q.

47. OA, OB and OC represent in magnitude and direction three vectors P, Q and R. If P + Q = 2R, prove that C is the midpoint of AB.

48. - i - j - k and - 2i + j + k these two vectors represent the two sides of a triangle. Determine the angle between these two sides and the length of the third side.

49. The position vectors of the three points A, B, C are (2i + 4j – k), (4i + 5j + k) and (3i + 6j - 3k) respectively. Show that the three points form a right-angled triangle.

50. In the trapezium ABCD, $\overrightarrow{AB}$ = 6i, $\overrightarrow{AD}$ = 3i + 4j, $\overrightarrow{DC}$ = 3i. Determine $\overrightarrow{BC}$, $\overrightarrow{BD}$ and $\overrightarrow{AC}$. What is the value of $\angle$BAD?

51. If a + b + c = 0, then prove that a x b = b x c = c x a.

52. Point A is situated on the circumference of a circle of diameter BC. Show that the angle BAC is a right angle.

53. A particle is in equilibrium under the simultaneous action of three forces. Prove that each bears a constant ratio with the sine of the angle between the other two.

54. If A is any vector, prove that A = (A . i) i + (A . j) j + (A . k) k.

55. Prove that the vectors A = i – 3j + 5k, B = 2i + j – 4k and C = 3i – 2j + k form a right-angled triangle.

56. Obtain the cosine law for triangles using the property of scalar product of two vectors.

57. The vectors 3i + 4j + 5k and 8i represent the two sides of a triangle. Find (i) the area of the triangle and (ii) a vector perpendicular to the plane of the triangle.

58. A force F has magnitude of 15 N. Direction of F is at 37^o from negative x-axis towards positive y-axis. Represent F in terms of $\hat{\imath}$ and $\hat{\jmath}$.

59. Obtain the magnitude of 2A - 3B if

$$A = \hat{\imath} + \hat{\jmath} - 2\hat{k} \text{ and } B = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

60. Work done by a force F on a body is W = F.s, where s is the displacement of body. Given that under a force F = $(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ N a body is displaced from position vector $r_1 = (2\hat{\imath} + 3\hat{\jmath} + \hat{k})$ m to the position vector $r_2 = (\hat{\imath} + \hat{\jmath} + \hat{k})$ m. Find the work done by this force.

61. Find the angle between two vectors $A = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $B = \hat{\imath} - \hat{k}$.

62. Prove that the vectors $A = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $B = \hat{\imath} + \hat{\jmath} + \hat{k}$ are mutually perpendicular.

63. Find a unit vector perpendicular to $A = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $B = \hat{\imath} - \hat{\jmath} + \hat{k}$ both.

64. Show that the vector $A = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ is parallel to a vector $B = 3\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.

65. A particle moves on a given line with a constant speed v. At a certain time, it is at a point P on its straight line path. O is a fixed point. Show that (OP x v) is independent of the position P.